# THERMODYNAMICS OF SOLUTIONS OF ETHANOL IN NONASSOCIATING COMPONENTS **\***

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#### ABSTRACT

The spectroscopic and thermodynamic properties of solutions of ethanol in nonassociating components can be described with good accuracy by the associated solution model of Nagata and Tamura (1982) based on mole fraction statistics.

### LIST OF SYMBOLS

A, B	alcohol and nonassociating component
$C_i, D_i$	constants of eqn. $(27)$
$G_{ij}$	coefficient as defined by $\exp(-\alpha_{ij}\tau_{ij})$
g	excess Gibbs free energy
$g_{ij}$	binary interaction parameter
h <sub>2</sub>	enthalpy of formation of dimer
$h_A$	enthalpy of hydrogen bond formation in $i$ -mer
$\frac{h_{AB}}{h^E}$	enthalpy of formation of chemical complex $A$ , $B$
	excess enthalpy
$K_{2}$	equilibrium constant of dimer formation
$K_{\mathcal{R}}$	equilibrium constant of open chain trimer formation
K	equilibrium constant of open chain <i>i</i> -mer formation, $i > 3$
$K_{cy}$	equilibrium constant for cyclization of open chain <i>i</i> -mer as defined by $\theta/i$ , $i > 4$
$K_{AB}$	equilibrium constant of formation of chemical complex $A_iB$
$\boldsymbol{P}$	total pressure
$P_i^s$	saturated vapor pressure of pure component $i$ .
R	gas constant

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### *Greek letters*



- *B*  coefficient as defined by eqn. (28)
- *7i*  activity coefficient of component i
- *0*  constant related to  $K_{\text{ex}}$
- chemical potential  $\mu$
- coefficient as defined by  $(g_{ii} g_{ii})/RT$  $\tau_{ii}$
- vapor phase fugacity coefficient of component i φ,
- vapor phase fugacity coefficient of pure component  $i$  at system  $\phi^{\rm S}$ temperature T and pressure  $P_i^s$

# *Subscripts*

- A,B alcohol and nonassociating component
- $A_1, A_i$ alcohol monomer and i-mer
- $A, B$ complex formation between alcohol i-mer and component B
- chem chemical
- phys physical
- 1,2 alcohol and nonassociating component

# *Superscripts*

- E excess
- L liquid
- s saturation
- \* pure alcohol

# INTRODUCTION

Stokes' chemical model [1] can reproduce well the spectroscopic and dielectric properties and the inflexions found in the partial thermodynamic properties of dilute solutions of ethanol in cyclohexane. The same model, with the addition of a solvation equilibrium, is able to account satisfactorily for the thermodynamic properties of ethanol in  $p$ -xylene [2]. The model works only at the dilute concentration range of ethanol. To cover the whole composition range, an improved model was developed [3], based on that of

Stokes [1], which has been used to correlate isothermal phase equilibrium data and excess enthalpy data of binary solutions of methanol and a solvating component and to predict phase equilibrium and excess enthalpy data of ternary solutions of methanol and solvating components. In this paper the new model is extended to the spectroscopic and thermodynamic properties of solutions of ethanol in nonassociating components.

#### ASSOCIATION MODEL

The model is similar to that previously used by Stokes, except that the NRTL equation [4] is used in place of the physical interaction term. To give a quantitative reproduction of the activity coefficients, excess enthalpies, and infrared spectroscopic behavior and to account for the initial decrease in the apparent dipole moment, the following was selected for the model in accordance with Stokes' assumptions.

(1) Open chains of any length are present according to stepwise reactions and three equilibrium constants,  $K_2$ ,  $K_3$  and  $K$ , are defined respectively as follows

$$
A + A = A_2 \t K_2 = x_{A_2}/x_{A_1}^2 \t (1)
$$

$$
A_2 + A = A_3 \t K_3 = x_{A_3}/x_{A_2}x_{A_1} \t (2)
$$

$$
A_{i} + A = A_{i+1} \t K = x_{A_{i+1}} / x_{A_{i}} x_{A_{i}} \t \text{ for all } i \geq 3
$$
 (3)

(2) Closed cyclic groups are also present and the first cyclic species is a pentamer. The cyclic groups are in equilibrium with the open chains of the same number of members and the equilibrium constant has the form

$$
A_i(\text{linear}) = A_i(\text{cyclic}) \quad K_{cy} = \theta / i = x_{A_i(\text{cyclic})} / x_{A_i(\text{linear})} \text{ for all } i > 4 \tag{4}
$$

where  $\theta$  is independent of i.

For the ethanol-solvating component systems, the model should include a solvation equilibrium between the terminal hydroxyl group of polymeric alcohol species and one solvating molecule (B).

$$
A_i(\text{linear}) + B = A_iB \quad K_{AB} = x_{A_iB}/x_{A_i}x_{B_i} \text{ for all } i \ge 1
$$
 (5)

The value of  $K_2K_3/K^2$  has to be independent of temperature according to the model. The temperature dependence of  $K_2$ ,  $K_3$ ,  $K$ ,  $\theta$  and  $K_{AB}$  should be given by, respectively

$$
\partial \ln K_2/\partial (1/T) = -h_2/R, \qquad \partial \ln K_3/\partial (1/T) = -(2h_A - h_2)/R
$$
  
\n
$$
\partial \ln K/\partial (1/T) = -h_A/R, \qquad \partial \ln \theta/\partial (1/T) = -h_A/R
$$
  
\n
$$
\partial \ln K_{AB}/\partial (1/T) = -h_{AB}/R
$$
 (6)

The chemical potential of the stoichiometric species is equal to that of the monomeric one [5].

$$
\mu_{\mathbf{A}} = \mu_{\mathbf{A}_1}, \mu_{\mathbf{B}} = \mu_{\mathbf{B}_1} \tag{7}
$$

Then, the chemical contribution terms of the excess Gibbs energy and activity coefficients for the ethanol (1)-solvating component (2) system are expressed by

$$
g_{\text{chem}}^{E} / RT = x_1 \ln(x_{A_1}/x_{A_1}^* x_1) + x_2 \ln(x_{B_1}/x_2)
$$
 (8)

$$
(\ln \gamma_1)_{\text{chem}} = \ln \left( x_{\mathbf{A}_1} / x_{\mathbf{A}_1}^* x_1 \right) \tag{9}
$$

$$
(\ln \gamma_2)_{\text{chem}} = \ln(x_{\text{B}_1}/x_2) \tag{10}
$$

The true mole fractions of the monomeric species,  $x_{A_1}$  and  $x_{B_1}$ , are related to the stoichiometric mole fraction  $x_1$ 

$$
x_1 = \left\{ \left( 1 + K_{AB} x_{B_1} \right) \left[ x_{A_1} + 2 K_2 x_{A_1}^2 + K_2 K_3 (3 - 2z) x_{A_1}^3 / (1 - z)^2 \right] + K_2 K_3 K^2 \theta x_{A_1}^5 / (1 - z) \right\} / S
$$
\n(11)

where  $z = Kx_A$ , and S is the stoichiometric sum given by

$$
S = (1 + K_{AB}x_{B_1})[x_{A_1} + 2K_2x_{A_1}^2 + K_2K_3(3 - 2z)x_{A_1}^3/(1 - z)^2]
$$
  
+ K<sub>2</sub>K<sub>3</sub>K<sup>2</sup>θx<sub>A\_1</sub><sup>5</sup>/(1 - z)  
+ K<sub>AB</sub>x<sub>B\_1</sub>[x<sub>A\_1</sub> + K<sub>2</sub>x<sub>A\_1</sub><sup>2</sup> + K<sub>2</sub>K<sub>3</sub>x<sub>A\_1</sub><sup>3</sup>/(1 - z)] + x<sub>B\_1</sub> (12)

The sum of the mole fractions of all chemical species present in the solution must be unity

$$
(1 + K_{AB}x_{B_i})[x_{A_1} + K_2x_{A_1}^2 + K_2K_3x_{A_1}^3/(1-z)]
$$
  
 
$$
- (K_2K_3\theta/K^3)[\ln(1-z) + z + z^2/2 + z^3/3 + z^4/4] + x_{B_i} = 1
$$
 (13)

The value of  $x_{A_1}^*$  is obtained from eqn. (14) at pure alcohol state

$$
\left[x_{\mathsf{A}_1}^* + K_2 x_{\mathsf{A}_1}^{*2} + K_2 K_3 x_{\mathsf{A}_1}^{*3}/(1-z^*)\right] - \left(K_2 K_3 \theta / K^3\right) \left[\ln(1-z^*) + z^* + z^{*2}/2 + z^{*3}/3 + z^{*4}/4\right] = 1
$$
 (14)

For the physical contribution term the NRTL equation is adopted [4], though Stokes used the Scatchard-Hildebrand equation

$$
g_{\text{phys}}^E / RT = x_1 x_2 [\tau_{21} G_{21} / (x_1 + x_2 G_{21}) + \tau_{12} G_{12} / (x_2 + x_1 G_{12})]
$$
(15)

$$
(\ln \gamma_1)_{\text{phys}} = x_2^2 \left[ \tau_{21} G_{21}^2 / (x_1 + x_2 G_{21})^2 + \tau_{12} G_{12} / (x_2 + x_1 G_{12})^2 \right]
$$
 (16)

$$
(\ln \gamma_2)_{\text{phys}} = x_1^2 \left[ \tau_{12} G_{12}^2 / (x_2 + x_1 G_{12})^2 + \tau_{21} G_{21} / (x_1 + x_2 G_{21})^2 \right]
$$
 (17)

where

$$
\tau_{21} = (g_{21} - g_{11})/RT \quad \tau_{12} = (g_{12} - g_{22})/RT \tag{18}
$$

$$
G_{21} = \exp(-\alpha_{21}\tau_{21}) \qquad G_{12} = \exp(-\alpha_{12}\tau_{12})
$$
\n(19)

Finally the excess Gibbs free energy and activity coefficients are given by the sum of both contributions

$$
g^{E} = g_{\text{chem}}^{E} + g_{\text{phys}}^{E}
$$
 (20)

$$
\ln \gamma_1 = (\ln \gamma_1)_{\text{chem}} + (\ln \gamma_1)_{\text{phys}} \tag{21}
$$

$$
\ln \gamma_2 = (\ln \gamma_2)_{\text{chem}} + (\ln \gamma_2)_{\text{phys}} \tag{22}
$$

The excess enthalpy of the solution is obtained by the sum of the chemical and physical contributions terms similarly

$$
h^{E} = h_{\text{chem}}^{E} + h_{\text{phys}}^{E}
$$
\n
$$
(23)
$$
\n
$$
h_{\text{chem}}^{E} = \left\{ \left( 1 + K_{AB} x_{B_{1}} \right) \left[ h_{2} K_{2} x_{A_{1}}^{2} + h_{A} K_{2} K_{3} x_{A_{1}}^{3} (2 - z) / (1 - z)^{2} \right] + h_{A} K_{2} K_{3} K^{2} \theta x_{A_{1}}^{5} / (1 - z) + h_{AB} K_{AB} x_{B_{1}} \left[ x_{A_{1}} + K_{2} x_{A_{1}}^{2} + K_{2} K_{3} x_{A_{1}}^{3} / (1 - z) \right] \right\} / S
$$
\n
$$
- x_{1} \left[ h_{2} K_{2} x_{A_{1}}^{*2} + h_{A} K_{2} K_{3} x_{A_{1}}^{*3} (2 - z^{*}) / (1 - z^{*})^{2} + h_{A} K_{2} K_{3} K^{2} \theta x_{A_{1}}^{*5} / (1 - z^{*}) \right] / S^{*}
$$
\n
$$
(24)
$$

$$
h_{\text{phys}}^{\text{E}} = \frac{\partial \left(g_{\text{phys}}^{\text{E}} / T\right)}{\partial (1/T)} = R x_1 x_2 \left\{ \frac{\tau_{21}^{\prime} G_{21}}{x_1 + x_2 G_{21}} + \frac{\tau_{12}^{\prime} G_{12}}{x_2 + x_1 G_{12}} - \alpha_{12} \left[ \frac{x_1 \tau_{21} G_{21} \tau_{21}^{\prime}}{\left(x_1 + x_2 G_{21}\right)^2} + \frac{x_2 \tau_{12} G_{12} \tau_{12}^{\prime}}{\left(x_2 + x_1 G_{12}\right)^2} \right] \right\}
$$
(25)

where  $S^*$  is the value of S at pure alcohol state and

$$
\tau'_{21} = \partial \tau_{21} / \partial (1/T) \quad \tau'_{12} = \partial \tau_{12} / \partial (1/T) \tag{26}
$$

The energy parameters are assumed to vary linearly with temperature

$$
g_{21} - g_{11} = C_1 + D_1(T - 273.15) \quad g_{12} - g_{22} = C_2 + D_2(T - 273.15) \tag{27}
$$

The infrared spectroscopic results furnish the ratio of the number of free

hydroxyl groups to the stoichiometric number of alcohol molecules,  $\beta$ 

$$
\beta = \frac{\sum_{i=1}^{\infty} x_{A_i(\text{linear})}}{\sum_{i=1}^{\infty} ix_{A_i(\text{linear})} + \sum_{i=5}^{\infty} ix_{A_i(\text{cyclic})}}
$$
  
= 
$$
\frac{x_{A_1} + K_2 x_{A_1}^2 + K_2 K_3 x_{A_1}^3/(1-z)}{x_{A_1} + 2K_2 x_{A_1}^2 + (K_2 K_3/K^3) z^3 [(3-2z)/(1-z)^2 + \theta z^2/(1-z)]}
$$
(28)

#### REPRESENTATION OF BINARY EXPERIMENTAL DATA

We use four equilibrium constants at 25°C and two enthalpies of hydrogen-bond formation for ethanol:  $K_2 = 40$ ,  $K_3 = 110$ ,  $K = 45$  and  $\theta = 85$ ;  $h_2 = -21.2$  kJ mol<sup>-1</sup> and  $h_A = -23.5$  kJ mol<sup>-1</sup>. The values of  $h_2$  and  $h_A$  are assumed to be temperature-independent. The values of the equilibrium constants are not the same as those given by Stokes [1]. It was found that the new model with a set of the equilibrium constants recommended by Stokes was unable to represent adequately the thermodynamic functions of the system ethanol-cyclohexane over the entire range of concentration. After many trials to find a best set of the equilibrium constants, the new values of the equilibrium constants are selected which reproduce well the spectroscopic data as well as the thermodynamic functions for the system ethanol-cyclohexane over the whole composition range. Table 1 lists the values of the solvation equilibrium constants and their enthalpies of complex formation used in data reduction. The enthalpies of complex formation were estimated by taking the difference between the value of enthalpy of dilution of ethanol in cyclohexane and that in a solvating component at  $25^{\circ}$ C as





TABLE 1



Fig. 1. Activity coefficients for ethanol(1)-cyclohexane(2). Calculated ( ). Experimental ( $\bullet$ , 6.7°C;  $\bullet$ , 25°C;  $\bullet$ , 45°C), data of Stokes and Adamson [6]. Experimental data at 6.7 and 45°C are shown at only very diluted concentrations of ethanol.

TABLE 2







Fig. 2. Molar excess enthalpies of ethanol(1)-cyclohexane(2). Calculated ( $$ mental ( $\blacktriangle$ , 6.7°C;  $\blacktriangleright$ , 25°C;  $\blacksquare$ , 45°C), data of Stokes and Adamson [6].  $-$ ). Experi-



Fig. 3. Activity coefficients for ethanol(1)- $p$ -xylene(2). Calculated (------). Experimental (O, |3.3°C; O, 25°C; A, 35°C; m, 45°C), data of Stokes and French [2]. Experimental data at 13.3, 35 and 45°C are shown at only very diluted concentrations of ethanol.









### TABLE 5

System	Temp.	No. of	Abs. arith. mean dev.		Ref.
	$(^{\circ}C)$	data points	Vapor mole fraction $(\times 1000)$	Pressure (mm Hg)	
Ethanol-			10.6		
benzene-	50	19	4,8	7.2	13
cyclohexane			8.4		
Ethanol-			7.3		
toluene-	50	19	4.5	3.4	15
cyclohexane			5.2		
Ethanol-			6.5		
$2$ -butanone-	25	33	4.6	0.9	12
benzene			4.5		
Ethanol-			11.0		
chloroform-	35	36	11.9	9.6	26
$n$ -hexane			12.3		

Predicted results for ternary vapor-liquid equilibria





Fig. 4. Molar excess enthalpies of ethanol(1)-  $p$ -xylene(2). Calculated ( $\rightarrow$  ). Experimental ( $\blacktriangle$ , 13.3°C;  $\blacktriangleright$ , 25°C;  $\blacktriangleright$ , 35°C;  $\blacktriangleright$ , 45°C), data of Stokes and French [2]. Note that the ordinate for the 13.3°C curve is displaced downwards by 1 kJ and those for the 35 and 45°C curves are displaced upwards by 1 and 2 kJ, respectively, to avoid overlap.

System	No. of data points	Abs. arith. mean dev. $(J \text{ mol}^{-1})$	Ref.
Ethanol-benzene-cyclohexane	18	14.1	27
Ethanol-toluene-cyclohexane		26.7	27
Ethanol- $p$ -xylene-cyclohexane	59	17.2	23
Ethanol-ethyl acetate-cyclohexane	51	26.8	24

TABLE 6

Predicted results for ternary excess enthalpies at 25°C

previously described [3]. Figures 1-4 show that the new model reproduces absolute values of activity coefficients and  $h^E/x_1x_2$  for the systems ethanol-cyclohexane and ethanol-p-xylene with marked success. In Figs. 1-4 the abscissa  $\sqrt{x_1}$  has been used to show the quality of fit in the region up to  $x_1 = 0.01$ . Table 2 gives the values of constants used in Figs. 1-4 to fit  $g^{E}/x_1x_2$  or  $h^{E}/x_1x_2$  data.

Figure 5 illustrates observed and calculated infrared spectroscopic data for the fraction of free-OH groups of ethanol molecules in two different solvents: (a) cyclohexane; (b) tetrachloromethane. Figure 5b shows clearly that a small but non-zero value of  $K_{AB}$  is necessary to obtain a good fit in ethanol(1)-tetrachloromethane(2), although the calculated curve deviates gradually from data at 15°C.

Vapor-liquid equilibrium data reduction was made using the equation

$$
\phi_i y_i P = x_i \gamma_i \phi_i^s P_i^s \exp[v_i^L (P - P_i^s) / RT]
$$
\n(29)

where  $P$  is the total pressure,  $P_i^s$  is the saturated vapor pressure of pure component i,  $\phi_i$  is the fugacity coefficient of component i at P,  $\phi_i^s$  is the fugacity coefficient at  $P_i^s$  and  $T$ ,  $y_i$  is the vapor mole fraction of component i, and  $v_i^L$  is the molar liquid volume of pure component *i*. The pure and cross second virial coefficients are estimated by the Hayden-O'Connell method [9] to calculate the fugacity coefficients. The Antoine equation is used to calculate the vapor pressures of pure components [10,11]. The nonrandom-

Fig. 5. Infrared spectroscopic data for fraction of free OH groups for (a) ethanol(1)-cyclohexane(2), and (b) ethanol(1)-tetrachloromethane(2). (a) Calculated (--, this work; ......, Stokes [1]). Experimental ( $\bullet$ , 15°C;  $\blacktriangle$ , 25°C;  $\Box$ , 35°C), data of Sassa and Katayama [7]. Note that the ordinates for 25 and 35°C curves are displaced upwards by 0.2 and 0.4, respectively, to avoid overlap.  $\blacksquare$ , Data at 35°C reduced by 10% to adjust for suggested extrapolation error. (b) Calculated ( $\longrightarrow$ ,  $K_{AB} = 1.4$  at 25°C;  $\longrightarrow$ ,  $K_{AB} = 0$ ). Experimental (. 15°C; A, 25°C; ■, 35°C), data of Sassa and Katayama [7]; ▼, data of Hoffmann [8] at 21.5°C. Note that the ordinates for the 21.5, 25 and 35°C curves are displaced upwards by 0.2, 0.4 and 0.6, respectively, to avoid overlap.









![](_page_14_Figure_1.jpeg)

![](_page_15_Figure_0.jpeg)

Fig. 8. Molar excess enthalpies for ethanol(1)-tetrachloromethane(2) at 35°C and for ethanol(1)-chloroform(2) at 25°C. Calculated (------). Experimental (O) ethanol-tetrachloromethane, data of Otterstedt and Missen [22]; (.) ethanol-chloroform, data of Nagata et al. [191.

-8oo~  $0.0$   $0.2$   $0.4$   $0.6$   $0.8$   $1.0$ Mole fraction of component 1

ness parameter of the NRTL equation,  $\alpha_{ij} = \alpha_{ji}$ , is taken as 0.3 for all binary systems studied here. Tables 3 and 4 present calculated results for selected binary systems. Figures 6-8 represent typical examples. The results demonstrate that the new model reproduces well the experimental data.

### PREDICTION OF TERNARY PROPERTIES FROM BINARY DATA

The model can be extended to predict ternary vapor-liquid equilibria and excess enthalpies by using binary parameters alone. Tables 5 and 6 give predicted results for representative systems. Reasonable agreement is ob-

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**tained between calculated results and experimental data, indicating that the new model is able to predict the ternary thermodynamic data of solutions of ethanol and nonassociating components.** 

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